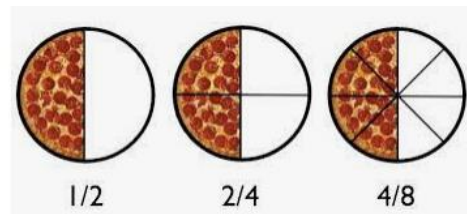

FRACTIONS AND DECIMALS

First, we need to understand what *equivalent fractions* are. Two fractions are **equivalent** if they represent the same number, even if they look different. For example, you know that we can reduce the fraction $\frac{3}{6}$ to $\frac{1}{2}$, meaning that the two fractions boil down to the same thing. The two fractions are therefore *equivalent*.

You can think of it this way: Whether you cut a pizza into 2 equal slices and eat 1 of them, or cut the pizza into 4 equal slices and eat 2 of them, or cut the pizza into 8 equal slices and eat 4 of them — you've eaten the same amount of pizza. Now we describe two ways to produce equivalent fractions.



3 Equivalent Fractions

❑ THE FUNDAMENTAL PRINCIPLE OF FRACTIONS

Dividing Top and Bottom by the Same Number

If you take any fraction, and divide both the numerator and denominator by the same non-zero number, the result is an **equivalent fraction**.

For example, this is the rule we might use to reduce a fraction to lowest terms. If we start with the fraction $\frac{6}{10}$, and then divide the top and bottom by 2, we get the equivalent fraction $\frac{3}{5}$:

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

We can apply the same principle even if we have no intention of reducing a fraction:

$$\frac{140}{220} = \frac{140 \div 10}{220 \div 10} = \frac{14}{22}$$

And the principle applies to Algebra as well:

$$\frac{aw}{bw} = \frac{aw \div w}{bw \div w} = \frac{a}{b}$$

The following section shows us how we can reverse this process.

Multiplying Top and Bottom by the Same Number

If you take any fraction, and then multiply both the numerator and denominator by the same non-zero number, the result is an equivalent fraction.

This is the process that allows us to “raise a fraction to higher terms.” For example, if we want to convert the fraction $\frac{4}{7}$ into a fraction whose denominator is 21, we would multiply top and bottom by 3:

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

The **3** comes from asking, “What do I have to multiply the **7** by, to get **21**?”

The principle can even be used to convert a fraction containing decimals to one whose denominator is a whole number, a process we’ll use with dividing decimals:

$$\frac{1.86745}{23.56} = \frac{1.86745 \times 100}{23.56 \times 100} = \frac{186.745}{2,356} \leftarrow \text{a whole number}$$

In symbols, the Fundamental Principle of Fractions can be written as follows:

For any non-zero number k , the fractions $\frac{a}{b}$ and $\frac{ak}{bk}$ are ***equivalent***; that is,

$$\frac{a}{b} = \frac{ak}{bk}$$

This principle means that you may either multiply or divide the top and the bottom of a fraction by the same non-zero number and end up with an *equivalent fraction*.

□ **ADDING AND SUBTRACTING FRACTIONS**

EXAMPLE 1:

A. Add: $2\frac{3}{4} + 5\frac{4}{5}$

The least common denominator (LCD) of 4 and 5 is 20:

$$\begin{array}{r} 2\frac{3}{4} = 2\frac{15}{20} \\ + 5\frac{4}{5} = 5\frac{16}{20} \\ \hline 7\frac{31}{20} = 8\frac{11}{20} \end{array}$$

B. Subtract: $18 - 5\frac{7}{10}$

Let's first line them up vertically:

$$\begin{array}{r} 18 \\ - 5\frac{7}{10} \\ \hline \end{array}$$

Since we can't subtract $\frac{7}{10}$ from nothing, we "borrow" one whole from the 18, and call it $\frac{10}{10}$. Then we can subtract:

$$\begin{array}{r} 17\cancel{18}\frac{10}{10} \\ - 5\frac{7}{10} \\ \hline 12\frac{3}{10} \end{array}$$

C. Subtract: $12\frac{5}{6} - 7\frac{8}{9}$

Let's first line them up vertically:

$$\begin{array}{r} 12\frac{5}{6} \\ - 7\frac{8}{9} \\ \hline \end{array}$$

The LCD (least common denominator) of 6 and 9 is 18:

$$\begin{array}{r} 12\frac{5}{6} = 12\frac{15}{18} \\ - 7\frac{8}{9} = 7\frac{16}{18} \\ \hline \end{array}$$

We can't subtract $\frac{16}{18}$ from $\frac{15}{18}$, so we borrow one whole from the 12, and call it $\frac{18}{18}$:

$$\begin{array}{r} 12\frac{15}{18} = 11\cancel{12}\frac{15}{18} + \frac{18}{18} \\ - 7\frac{16}{18} \\ \hline \end{array}$$

So we now have a subtraction problem we can finish:

$$\begin{array}{r}
 11\frac{33}{18} \quad (\text{the result of adding } \frac{15}{18} \text{ and } \frac{18}{18}) \\
 - 7\frac{16}{18} \\
 \hline
 4\frac{17}{18}
 \end{array}$$

□ **MULTIPLYING AND DIVIDING FRACTIONS**

A.

$$\frac{2}{3} \cdot \frac{5}{9} = \frac{2 \cdot 5}{3 \cdot 9} = \frac{10}{27}$$

B.

$$4\frac{1}{7} \times 14 = \frac{29}{7} \times \frac{14}{1} = \frac{29}{7} \times \frac{2 \cdot 7}{1} = \frac{29}{\cancel{7}} \times \frac{2 \cdot \cancel{7}}{1} = \frac{29 \times 2}{1} = 58$$

C. $\frac{4}{5} \div \frac{5}{7} = \frac{4}{5} \times \frac{7}{5} = \frac{28}{25} = 1\frac{3}{25}$

D.

$$6 \div 7\frac{4}{5} = \frac{6}{1} \div \frac{39}{5} = \frac{6}{1} \times \frac{5}{39} = \frac{2 \cdot 3}{1} \times \frac{5}{3 \cdot 13} = \frac{2 \cdot \cancel{3}}{1} \times \frac{5}{\cancel{3} \cdot 13} = \frac{10}{13}$$

E.

$$\frac{2\frac{2}{3}}{11\frac{4}{9}} = 2\frac{2}{3} \div 11\frac{4}{9} = \frac{8}{3} \div \frac{103}{9} = \frac{8}{3} \cdot \frac{9}{103} = \frac{\cancel{8} \cdot \cancel{9} \times 3}{\cancel{3} \cdot 103} = \frac{24}{103}$$

❑ DECIMAL OPERATIONS

Adding: The key to adding decimals is threefold: First, realize that a whole number has its decimal point at the right of the number, as in:

$$234 = 234.$$

Second, line up the decimal points, and then add as usual. Third, bring the decimal point straight down into the sum.

$$45 + 9.76 + 13.8 \longrightarrow \begin{array}{r} 45. \\ 9.76 \\ 13.8 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 45.00 \\ 9.76 \\ 13.80 \\ \hline 68.56 \end{array}$$

Subtracting: Except for the unpleasant prospect of borrowing, the rules for subtracting decimals are identical to those for adding decimals. Be sure that the larger number is on top.

Multiplying: To multiply two decimals, line up the numbers flush on the right (just like whole-number multiplying -- putting the number with more digits on top) and multiply as usual. Now count up the total number of digits to the right of the decimals being multiplied, and make sure the answer has that many digits to the right of the point.

$$2.3 \times 0.009 \longrightarrow \begin{array}{r} 0.009 \\ \times 2.3 \\ \hline 27 \\ 180 \\ \hline 0.0207 \end{array} \quad \begin{array}{l} \leftarrow 3 \text{ digits to the right of the point} \\ \leftarrow 1 \text{ digit to the right of the point} \\ 3 + 1 = 4 \\ \leftarrow 4 \text{ digits to the right of the point} \end{array}$$

Dividing: There are two cases to consider:

Case 1: The divisor (the outside number) is a whole number: Bring the decimal point in the dividend (the inside number) straight up, and divide as usual. Rounding is a separate issue and depends upon the original problem and your teacher's instructions.

$$\begin{array}{r} 5.9 \\ 23 \overline{)135.7} \\ \underline{115} \\ 207 \\ \underline{207} \\ 0 \end{array}$$

Case 2: The divisor (the outside number) is not a whole number:

Let's look at the division problem:

$$0.19 \overline{)1.732}$$

Here's an approach consistent with the ideas presented in this chapter. Rewrite the problem as a fraction (inside number goes on the top);

$$\frac{1.732}{0.09}$$

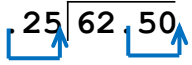
Multiply top and bottom by whatever power of 10 turns the denominator into a whole number — in this case, that would be 100:

$$\frac{1.732}{0.09} = \frac{1.732 \times 100}{0.09 \times 100} = \frac{173.2}{9}$$

Now divide as explained in Case 1.

Here's the standard approach: Move the decimal point in the divisor to the right as many places as necessary to convert it to a whole number (that is, all the way to the right of the divisor). Now move the decimal point in the dividend to the right the same

number of places, and bring the decimal point straight up. Finally, divide as usual, and round according to the rules of the problem or the class. Check out the example:

$0.25 \overline{)62.5}$	Divide 62.5 by 0.25
$0.25 \overline{)62.50}$ 	Move each decimal point 2 places to the right
$25 \overline{)6250.}$	The final answer is 250

❑ **CONVERTING BETWEEN FRACTIONS AND DECIMALS**

Fraction → Decimal

To convert a **fraction to a decimal**, divide the numerator by the denominator. [Recall the Top-Inside-Left rule for division: The Top number of the fraction always goes Inside the division house.]

$$\frac{13}{20} = 20 \overline{)13} = 0.65$$

$$\frac{65}{50} = 50 \overline{)65} = 1.3$$

$$\frac{2}{3} = 3 \overline{)2} = 0.666666\dots \text{ forever and ever,}$$

which we might round to 0.67 or 0.667, or to whatever precision the problem requires.

Decimal → Fraction

To convert a **decimal to a fraction**, write the decimal number over the appropriate power of 10 (determined by the place value of the last digit), and reduce the fraction.

$$0.9 = \frac{9}{10} \quad (\text{the 9 lies in the tenths place})$$

$$0.125 = \frac{125}{1,000} = \frac{1}{8} \quad (\text{the 5 lies in the thousandths place})$$

$$2.0005 = 2\frac{5}{10,000} = 2\frac{1}{2,000} \quad (\text{the 5 is in the ten-thousandths place})$$

Solutions

*“Not with what we have,
but with what we enjoy,
do we find our true abundance.”*

Tibetan quote